Mathematical Reasoning: Practice Exam

https://sites.google.com/view/snedmaths/

AS 91947

3

1.4 Demonstrate mathematical reasoning

Total

0	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24
not achieved		nearly achieved	low achieved	high achieved	low merit	high merit	low excellence	high excellence
NOT ACHIEVED			ACHIEVED		MERIT		EXCELLENCE	
0-6			7-12		13-18		19-24	

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$A = \pi(r+2)^2 - \pi r^2$ $= \pi(r^2 + 4r + 4 - r^2)$ $= 4\pi r + 4\pi$ $= 4(\pi r + \pi)$ $= \pi(4r + 4)$ $= 4\pi(r + 1)$	**must** eliminate πr^2 term any reasonable expression		
(b)	$V = \frac{1}{3}Ah$ $h = 3 \times 1000 \div 250$ $h = 12$ cm	12 cm **units required**		
(c)	$A = (10 - x)(15 - x) = 150 - 25x + x^2$ $x^2 - 25x + 150 = 84$ $x^2 - 25 + 66 = 0$ $(x - 3)(x - 22) = 0$ $x = 3$ or $x = 22$ so the strip is **3 metres** wide [not 22 since this strip is wider than the field]	rearrange equal to zero	**single** solution algebra required	
(d)	elevation angles of rope: to P: $x = \tan^{-1}\frac{8}{10} = 38.66°$ to Q: y= $\tan^{-1}\frac{12}{10} = 50.19°$ So $z = 180 - x - y = 91.15°$	either angle x or y	angle z	
(e) (i)	$\frac{1}{3}\pi R^2 H : \frac{1}{2}\frac{4}{3}\pi R^3 : \pi R^2 H$ $\frac{1}{3}\pi R^3 : \frac{2}{3}\pi R^3 : \pi R^3$ $\frac{\pi}{3} : \frac{2\pi}{3} : \pi$ $1 : 2 : 3$	expression with no H	expression eliminating R^3	integers

(e) (ii)	$\pi r\left(r+\sqrt{r^2+h^2}\right)=S$ $r+\sqrt{r^2+h^2}=\frac{S}{\pi r}$ $\sqrt{r^2+h^2}=\frac{S}{\pi r}-r$ $r^2+h^2=\left(\frac{S}{\pi r}-r\right)^2$ $h^2=\left(\frac{S}{\pi r}-r\right)^2-r^2$ $h^2=\frac{S^2}{\pi^2 r^2}-\frac{2S}{\pi}+r^2-r^2$ $h^2=\frac{S^2}{\pi^2 r^2}-\frac{2S}{\pi}$ $h=\sqrt{\frac{S^2}{\pi^2 r^2}-\frac{2S}{\pi}}$ $h=\sqrt{\frac{S}{\pi}\left(\frac{S}{\pi r^2}-2\right)}$	isolate square root term	any expression for h^2	either **simplified** expression for h

Each Question

no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8

Evidence Statement

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$h^2 = 3^2 - 1.1^2$ $h = 2.79$ metres			
(b)	$(5x - 1)(x - 3) \leq 0$ parabola intersects x-axis at $x = 0.2$ and $x = 3$ the parabola is below the x-axis when $0.2 \leq x \leq 3$	factorised or roots found (any method)	correct interval	
(c)	$m = \frac{\text{rise}}{\text{run}} = \frac{-2-4}{5-1} = \frac{-6}{4} = -1.5$ $y = -1.5x + c$ substitute a point $4 = -1.5 + c$ $c = 5.5$ $y = -1.5x + 5.5$	equation of line		
(d)	distance to car $x = 18 \div \tan 12 = 84.68$ m new distance to car $y = 184.68$ m new angle of depression $\tan A = \frac{18}{184.68}$ $A = \tan^{-1}\left(\frac{18}{184.68}\right) = 5.57°$	distance to car	correct angle	
(e)	$2^{x-1} = 4^{x+1}$ $2^{x-1} = (2^2)^{x+1}$ $2^{x-1} = 2^{2x+2}$ $x - 1 = 2x + 2$ $-3 = x$ $x = -3$	changes $4 = 2^2$ OR without algebra $x = -3$	equates powers	algebraically gets $x = -3$
(f)	$A = (x + 2)(x + 3) - \frac{1}{2}(x + 1)(x + 2)$ $20 = x^2 + 5x + 6 - 0.5(x^2 + 3x + 2)$ $20 = x^2 + 5x + 6 - 0.5x^2 - 1.5x - 1$ $20 = 0.5x^2 + 3.5x + 5$ $x^2 + 7x + 10 = 40$ $x^2 + 7x - 30 = 0$ OR $0.5x^2 + 3.5x - 15 = 0$ $(x + 10)(x - 3) = 0$ reject $x = -10$, the dimensions have $x = 3$ **the metal sheet is 5 metres by 6 metres**	expands both expressions correctly	forms equation equal to zero	dimensions of sheet, 5×6

Evidence Statement

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$= \dfrac{(x+1)(x+5)}{(x+1)(x+7)} = \dfrac{x+5}{x+7}$	some working to answer		
(b)	$u^2 = v^2 - 2as$ $u = \sqrt{v^2 - 2as}$	CAO		
(c)	$x + y = 52$ $y = 2x + 7$ substitution $x + 2x + 7 = 52$ $3x = 45$ $x = 15$ $y = 2 \times 15 + 7$ $y = 37$	forms equations	finds both numbers	
(d)	The differences are 4,6,8 The second differences are 2,2,2 The pattern starts with n^2: 1,4,9,16 the remainder is 2,3,4,5 equation for remainder $n + 1$ the rule for the nth term is $n^2 + n + 1$	second differences OR correct answer no working	correct answer, algebraic working	
(e)	red: $y = a(x-1)(x-3)$ using roots using the y-intercept, $3 = a(0-1)(0-3)$ so $a = 1$ and the equation is $y = (x-1)(x-3)$ blue: $y = a(x-1)(x-2)$ using roots using the y-intercept, $-2 = a(0-1)(0-2)$ so $a = -1$ and the equation is $y = -(x-1)(x-2)$ each vertex is half-way between the intercepts, so the vertices are red: $(2, -1)$ blue: $(1.5, 0.25)$ The distance between these points is $d_x = 0.5$ and $d_y = 1.25$ so $d = \sqrt{0.5^2 + 1.25^2} = \sqrt{1.8125} = 1.346$	both equations	both vertices	distance
(f)	Intersections $(x-1)(x-3) = -(x-1)(x-2)$ $x^2 - 4x + 3 = -x^2 + 3x - 2$ $2x^2 - 7x + 5 = 0$ $(x-1)(2x-5) = 0$ $x = 1$ or $x = 2.5$ points $(1,0)$ and $(2.5, -0.75)$ $m = \frac{\text{rise}}{\text{run}} = -\frac{0.75}{1.5} = -0.5$ $y = -0.5x + c$ substitute either point (using $(1,0)$): $0 = -0.5 \times 1 + c$ so $c = 0.5$ line is $y = -0.5x + 0.5$	intersection x-coordinates	gradient of line	equation of line