

**AS 91947**

## **1.4 Demonstrate mathematical reasoning (5 credits)**

You should attempt ALL the questions in this booklet.

The resource booklet 91947R should be with this booklet.

Show ALL working.

An approved calculator is allowed for this assessment.

Achievement	Achievement with Merit	Achievement with Excellence	Score	Grade
Demonstrate mathematical reasoning.	Demonstrate mathematical reasoning with relational thinking.	Demonstrate mathematical reasoning with extended abstract thinking.		

Grading information

Each Question

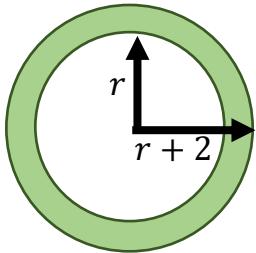
no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8

Total

0	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24
not achieved	nearly achieved	low achieved	high achieved	low merit	high merit	low excellence	high excellence	
NOT ACHIEVED			ACHIEVED			MERIT		EXCELLENCE
0-6			7-12			13-18		19-24

**QUESTION ONE**

(a) A circular track has an inner radius of  $r$  metres and a constant width of 2 metres all the way around.



Write an expression for the area of the track in terms of  $r$  and simplify it.

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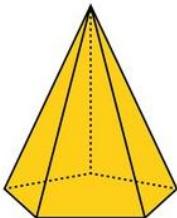
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(b) A pyramid with base area  $250 \text{ cm}^2$  is shown below.



The volume of the pyramid is one litre ( $1000 \text{ cm}^3$ ). Find the height of the pyramid.

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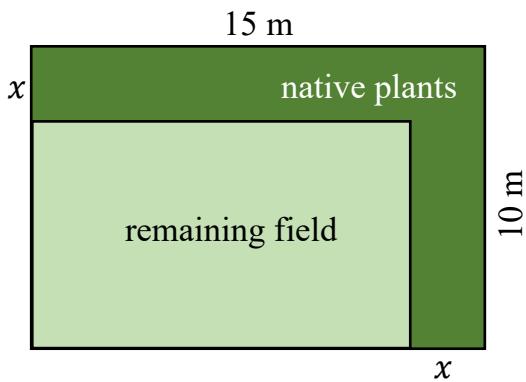
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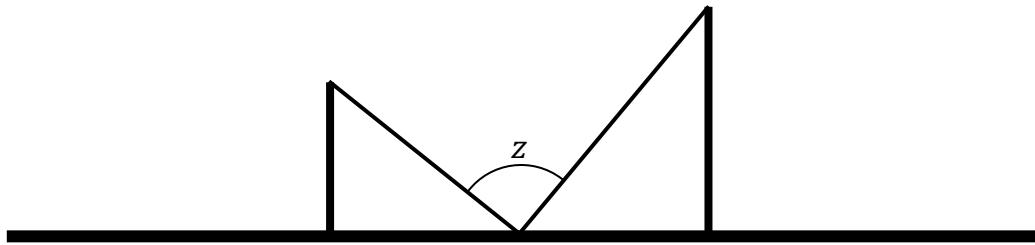
(c) A rectangular field has two strips of native plants along the edges as shown.

Show that the areas of the remaining field is  $A = x^2 - 25x + 150$ , and hence find the value of  $x$  when the area of the remaining field is 84 square metres.



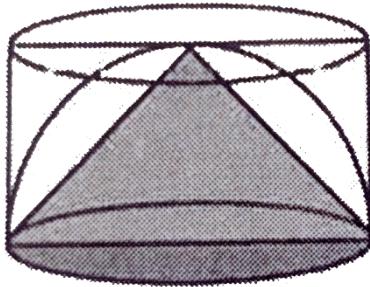
(d) Two vertical poles stand on level ground 20 m apart.  
The poles are 8m high and 12m high.

A stake is placed on the ground half-way between the poles, and a rope is tied between the stake and the tops of the two poles as shown.



Find the angle between the ropes at the stake,  $z$ .

(e) A cone fits perfectly inside a hemisphere which fits perfectly inside a cylinder, all of the same base radius  $R$ . The all have the same height,  $H = R$ .



(i) Form and simplify the ratio of the **volumes** so that the terms are integers.

volume of cone : volume of hemisphere : volume of cylinder

$$a:b:c$$

(ii) Another equation for cones gives the surface area,  $S = \pi r(r + \sqrt{r^2 + h^2})$ .

Rearrange this equation to find an expression for  $h$  in terms of  $r$  and  $S$ .

Expand and simplify your answer if possible.

## QUESTION TWO

(a) A 3 metre long ladder leans against a wall.

The base of the ladder is 1.1 metres away from the wall.

Calculate how high up the wall the ladder reaches.



(b) Solve the inequality  $5x^2 - 16x + 3 \leq 0$ .

(c) A straight line passes through the  $(x, y)$  points  $(1, 4)$  and  $(5, -2)$ .

Find the equation of the line in the form  $y = mx + c$ .

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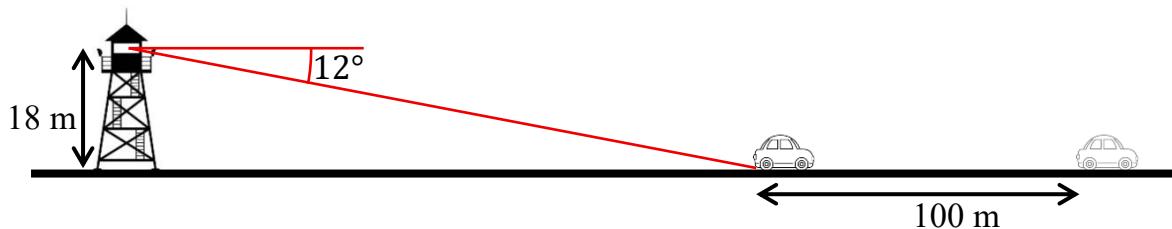
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(d) From a lookout tower with height 18 metres, the angle of depression to a car on a level road is  $12^\circ$ .

Find the new angle of depression when the car moves a further 100m away from the tower.

The diagram below represents the situation but is not to scale.



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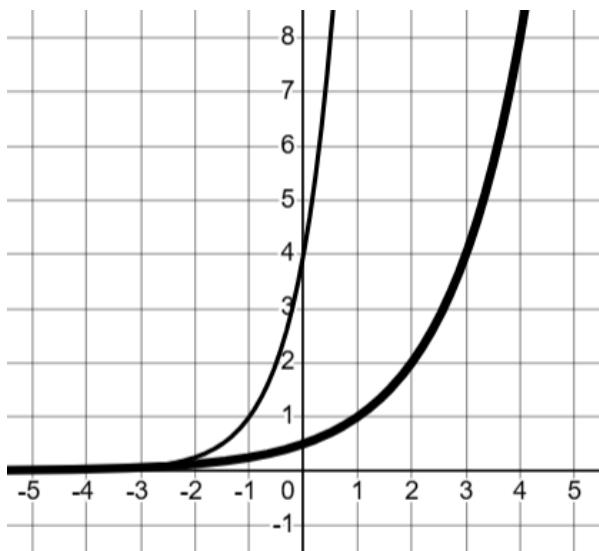
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(e) The two curves given are shown on the axes below.

$$y = 2^{x-1}$$

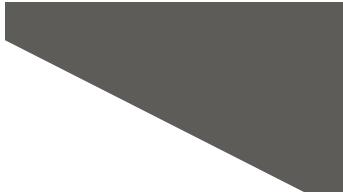
$$y = 4^{x+1}$$



Use algebraic methods to find the coordinates where the curves intersect.

(f) A rectangular sheet of metal with side lengths  $x + 2$  by  $x + 3$  metres has a triangular corner removed.

The triangular corner has side lengths  $x + 1$  and  $x + 2$  metres.



Find the dimensions of the sheet of metal if the total shaded area is 20 square metres.

**QUESTION THREE**

(a) Simplify

$$\frac{x^2 + 6x + 5}{x^2 + 8x + 7}$$

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(b) Rearrange  $v^2 = u^2 + 2as$  to make  $u$  the subject.

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(c) Two numbers add to 52.

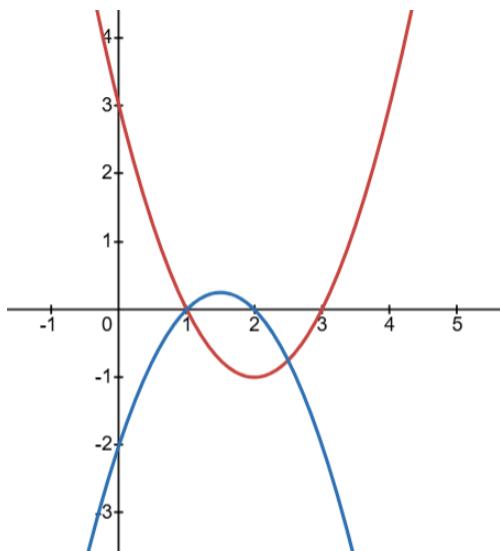
The larger number is 7 more than twice the smaller number.

Find both numbers.

(d) The first four terms in a **quadratic** pattern are 3, 7, 13, 21.

Use algebra to find a rule for the  $n$ th term.

(e) Two parabolas are shown below.



(i) For each parabola, find its equation and vertex.

Hence, find the distance between the vertices.

'(ii) Using the two parabolas from the previous question, the shape in the middle is used to make part of a company logo. A line between the intersection points is added.

Find an equation for the line.

