MATH 1.1 – Experimental Probability Investigation Exemplar
Mr Wills

Problem Statement

This report investigates the probability that Mr Wills has 7 good nights of sleep in a week based on probabilities from Mr Wills' record of sleep from 2020 to 2023.

Background: Sleep is important for our wellbeing. It is essential for body repair and retaining memories. Mr Wills has recorded data for the last three years and tells us he thinks he gets a good night's sleep 70% of the time. He wants to know how likely he is to get a good night of sleep for a whole week.

Hypothesis: I think Mr Wills gets a week of good sleeps more than half of the time. If he has a good night's sleep 70% of the time, then he probably has good sleeps for a full week most of the time.
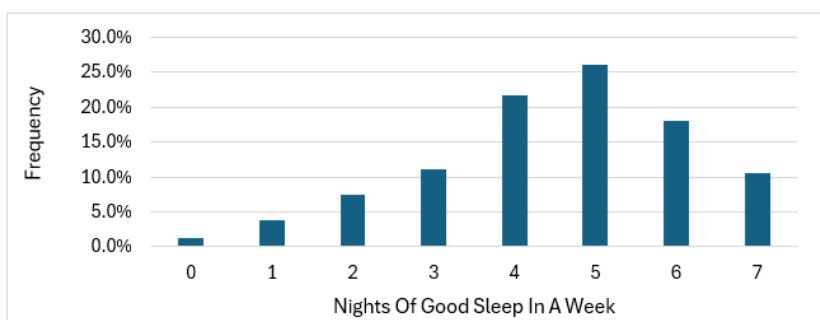
Plan

Mr Wills recorded whether a sleep was good or not every day for the last three years. We used Excel to process this data into a two-way-table, plotting the frequency of the number of good sleeps each week being 0-7 good nights of sleep. These were then plotted as a relative frequency by dividing by the total weeks tracked (161) which was much more than the minimum of 30 trials which are required. From these values we calculated a Mean, Median, Mode, and Standard Deviation, and made a Histogram. A simulation using Mr Wills's estimated 70% good sleeps was made using Plinko Probability from PHET.

Sources of Variation: Measurement variation may have occurred if Mr Wills has not been consistent in what counts as a "Good Sleep". He could have controlled this better by ensuring a "Good Sleep" had a clear definition like "More than 7.5 hours and no more than 30 minutes tossing and turning" or by recording using a smart watch.
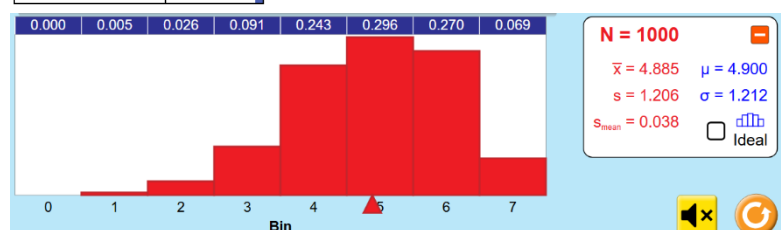
Mr Wills is only one person so these results may not be applicable to others outside of Mr Wills. We could have used a wider range of people and averaged the results to find if these results are consistent across many people.

Data

Outcomes	Frequency	Relative Frequency
0	2	1%
1	6	4%
2	12	7%
3	18	11%
4	35	22%
5	42	26%
6	29	18%
7	17	11%
Total Weeks Tracked	161	100%

Parameter	Value
Mean	4.51553
Median	5
Mode	5
SD	1.61555

| 0.000 | 0.005 | 0.026 | 0.091 | 0.243 | 0.296 | 0.270 | 0.069 |

N = 1000
x̄ = 4.885   μ = 4.900
s = 1.206   σ = 1.212
s_mean = 0.038   ☐ Ideal

Analysis

Clusters: There are no apparent clusters in this dataset.

Unusual Points: It would be unusual to get no good sleeps in a whole week which occurred 2 times out of 161 but as this is not far away from the rest of our data these points cannot be considered unusual.

Centre: The median number of good night's sleep that Mr Wills had in a week (out of 7) was 5. The average number of good nights of sleep Mr Wills got in a week was 4.52. The mode - most frequent number of good nights of sleep Mr Wills got in a week was also 5.

Spread: The standard deviation, the average amount each value deviates from the mean is 1.62 good nights of sleep per week. This means the probability of having 7 goods nights of sleep in a week is $(7-4.52)/1.62 = 1.53$ standard deviations above the mean.

Shape: The distribution appears to show a slight left skew. This is also evident as the mean is 0.48 nights of sleep per week below the median.

Patterns: There are no apparent additional patterns present.

Simulation: The simulation had the parameters of 70%, and 8 bins (labelled 0-7). The results differed from the observed relative frequencies. The median and mode were the same at 5 but the mean of 4.885 was greater by 0.37 nights per week. The simulation SD was slightly below our calculated SD by 0.4 nights per week. The greatest differences in our simulation were 7 good nights of sleep occurring only 6.7% of the time compared to 10.6% of the time in real life. The overall shape of the distribution is the same but real life shows greater spread.

Conclusion

This report concludes that the probability that Mr Wills has 7 good nights of sleep in a week is 10.6% while our simulation showed this probability is 6.9% based on probabilities from Mr Wills' record of sleep from 2020 to 2023.

Enough Data / Reliable Data: We can assume our conclusion is reliable because our dataset contained a sufficient amount of data (161 weeks) but we cannot assume the results are reliable as, although Mr Wills may be trustworthy, we cannot assume his results will reflect the results of others.

Other Improvements: If I were to do this experiment again, I would make sure a clear measure of a "Good Night's Sleep" was agreed on and I would record data in many more people so we could use the results for the general population.