# Mathematical Reasoning: Practice Exam

https://sites.google.com/view/snedmaths/

## AS 91947

5

### 1.4 Demonstrate mathematical reasoning

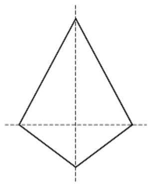
Evidence Statement

| Q1 | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
|---|---|---|---|---|
| (a) | $0.1x + 3 > 11$<br>$0.1x > 8$<br>$x > 80$ | correct inequality (not $\geq$) | | |
| (b) | $m = \frac{1-3}{5-0} = -0.4$<br>$y = -0.4x + 3$ | correct equation | | |
| (c) | $\frac{1}{2}\left(x + (x+3)\right) \times 7.5 = 69$<br>$2x + 3 = 69 \div 3.75$<br>$2x = 15.4$ so $x = 7.7$ m<br>$d = \sqrt{7.5^2 + 3^2} = 8.078$<br>$P = 7.5 + x + x + 3 + d = 33.98$ m | correct $x = 7.7$ | correct perimeter $P = 33.98$ | |
| (d) | using the $y$-axis intercept $(0,2)$<br>[could also use other points to find $p$ and $q$]<br>$2 = 2^{0+p}$ so $p = 1$ the equation is $y = 2^{x+1}$<br>$2 = 0^2 + 0 + q$ so $q = 2$ the equation is<br>$y = x^2 + x + 2$<br>substituting $x = 1$ and $x = 2$ into both<br>$2^{1+1} = 1^2 + 1 + 2 = 4$ giving the point $(1,4)$<br>$2^{2+1} = 2^2 + 2 + 2 = 8$ giving the point $(2,8)$ | $p = 1$ AND $q = 2$ | checking the other points | |
| (e) | Call one angle $x$ and the other is $90 - x$. The sides beside the angle $x$ are the hypotenuse $h$ and adjacent $a$. This side $a$ is opposite angle $90 - x$.<br>$\cos x = \frac{a}{h}$<br>$\sin(90 - x) = \frac{a}{h}$<br>Whatever the side lengths, the trig ratios will be equal | starts investigation by finding sides of a right triangle with angles 43 and 47 | demonstrates the claim is true with an example with different angles | 'proof' is generalised for any triangle |
| (f) | The surface area is two ends and the rectangle,<br>$2A + PL = 640$<br>$24L = 600$<br>$L = 25$ mm<br>$V = AL = 20 \times 25 = 500\text{ mm}^3$ | correct area equation | some algebra finding $L = 25$ mm | $V = 500\text{ mm}^3$ |

Evidence Statement

| Q2 | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
| --- | --- | --- | --- | --- |
| (a) | $3x - 11 = \frac{1}{2}x + 4$<br>$2.5x = 15$<br>$x = 6$<br>the intersection point is $(6,7)$ | point $(6,7)$<br><br>note: $x = 6$ is **insufficient** | | |
| (b) | converting to cm<br>$V = \left(\frac{1}{2} \times 4.5 \times 4.5\right) \times 0.9 = 9.1125$<br>mass $= 9.1125 \times 2.525 = 23.009$<br>$= 23$ grams | 23 grams require units | | |
| (c) | height above eye-line to top of flagpole<br>$h = 25 \tan 18.5° = 8.3649$ metres<br>total height $H = 8.3649 + 1.64$<br>$= 10.0049$ metres<br>percentage $\frac{1.64}{10.0049} \times 100\% = 16.39\%$ | 8.36 metres | 16% | |
| (d) | $x^2 + x^2 + 1.7^2 = 1.9^2$<br>$2x^2 = 3.61 - 2.89 = 0.72$<br>$x^2 = 0.36$<br>Volume $V = x^2L = 0.6^2 \times 1.7$<br>$= 0.612$ cubic metres<br>($x = 0.6$ intermediate step not required) | correct working to $x^2 = 0.36$ | correct volume | |
| (e) | calculating distances in $x$- and $y$-directions:<br>$AD_x = 9.85 \sin 24 = 4.01$<br>$AD_y = 9.85 \cos 24 = 9.00$<br>$AB_x = 5 \sin(360 - 307) = 3.99$<br>$AB_y = 5 \cos 53 = 3.01$<br>$BC_x = 7.07 \sin 8 = 0.98$<br>$BC_y = 7.07 \cos 8 = 7.00$<br>$CD_x = AB_x + AD_x - BC_x = 7.02$ km<br>$CD_y = AB_y + BC_y - AD_y = 1.01$ km<br>distance $CD = \sqrt{7.14^2 + 1.03^2} = 7.09$ km<br>angle $\alpha = \tan^{-1}\left(\frac{1.01}{7.02}\right) = 8.2°$<br>so the bearing from $C$ to D is 098.2° | three useful distances found | EITHER correct angle $\alpha = 8.2°$<br>OR correct distance | correct bearing 098º AND distance 7.09 km |
| (f) | $b^{-8}a^xb^x = \frac{a^x}{b^x}$<br>$a^xb^{x-8} = a^xb^{-x}$<br>$b^{x-8} = b^{-x}$<br>equating powers of $b$: $-8 + x = -x$<br>$2x = 8$<br>$x = 4$ | CAO<br><br>OR expanding correctly | collecting terms, eliminating $a^x$ factors | algebraic working to answer |

Evidence Statement

| Q3 | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
|---|---|---|---|---|
| (a) | side length $921 \div 4 = 230.25$<br>volume $V = \frac{1}{3} \times 23.025^2 \times 138.5$<br>$= 2{,}447{,}528 = 2.45 \times 10^6$ m$^3$ | correct, with units, to at least 3 s.f. | | |
| (b) | the $n$th term is $T_n = 4n + 3$<br>$T_{16} = 4 \times 16 + 3 = 67$ | CAO | | |
| (c) | dark region $A_\text{D} = \frac{1}{2}(2+7) \times 3 = 13.5$ cm$^2$<br>light region $A_\text{L} = \frac{1}{2}(2+5) \times 5 = 17.5$ cm$^2$<br>ratio $13.5 : 17.5 = 27 : 35$ | both areas | working shown for ratio | |
| (d) | $2x^2 + x = 10$<br>$2x^2 + x - 10 = 0$<br>$(x-2)(2x+5) = 0$<br>$x = 2$ or $x = -2.5$ | expanded and rearranged | algebraic working for both solutions | |
| (e) | draw a horizontal line<br>bottom triangles have angles $37°, 53°, 90°$<br>top triangles have angles $28°, 62°, 90°$<br>side lengths of bottom triangle<br>$10 \sin 53 = 7.99$ and $10 \cos 53 = 6.02$<br>7.99 is also a side of top triangle<br>hypotenuse $h = 7.99 \div \sin 28 = 17.01$<br>vertical $v = 7.99 \div \tan 28 = 15.02$<br>perimeter $P = 2 \times 17.01 + 2 \times 10 = 54.02$<br>area $A = 2 \times \frac{1}{2} \times 21.04 \times 7.99 = 168.02$ | **key** length 7.99 cm found<br> | perimeter OR area | perimeter AND area |
| (f) | $y = kx^2 + 3$ passes through $(3,6)$<br>$6 = k \times 3^2 + 3$ so $9k = 3$ and then $k = \frac{1}{3}$<br>Put $x = 1.5$ into $b = \frac{1}{3} \times 1.5^2 + 3 \Rightarrow b = 3.75$<br>Put $x = 2.5$ into $c = \frac{1}{5} \times 2.5^2 + 1 \Rightarrow c = 2.25$<br>$B = (1.5, 3.75)$ and $C(2.5, 2.25)$<br>halfway would be the average (midpoint)<br>$\frac{A+C}{2} = \left(\frac{0.5+2.5}{2}, \frac{5.25+2.25}{2}\right) = (1.5, 3.75)$<br>so $B$ is indeed halfway between $A$ and $C$ | finds ONE of<br>$k = \frac{1}{3}$<br>$b = 3.75$<br>$c = 2.25$ | vertical separation between points found OR attempt to find midpoint | $B$ shown to be midpoint [other methods possible] |

Each Question

| no attempt | relevant attempt | 1u | 2u | 3u | 1r | 2r | 1t | 2t |
|---|---|---|---|---|---|---|---|---|
| N0 | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |

Total

| 0 | 1-3 | 4-6 | 7-9 | 10-12 | 13-15 | 16-18 | 19-21 | 22-24 |
|---|---|---|---|---|---|---|---|---|
| not achieved | | nearly achieved | low achieved | high achieved | low merit | high merit | low excellence | high excellence |
| NOT ACHIEVED | | | ACHIEVED | | MERIT | | EXCELLENCE | |
| 0-6 | | | 7-12 | | 13-18 | | 19-24 | |