

Mathematical Reasoning: Practice Exam

<https://sites.google.com/view/snedomaths/>

AS 91947

4

1.4 Demonstrate mathematical reasoning

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$h = 2d = 2 \times 2r$ so $V = \pi r^2 h$ gives $V = 4\pi r^3$	allow CAO		
(b)	$A = (40 - 2x)(60 - 2x) \\ = 2400 - 200x + 4x^2$	any correct form		
(c)	hypotenuse $\sqrt{3^2 + 4^2} = 5$ (twice), hypotenuse $\sqrt{1^2 + 1^2} = \sqrt{2}$ perimeter $P = 5 + 5 + \sqrt{2}$ $P = 11.414$	any hypotenuse with calculation shown	$P = 11.414$	
(d)	distances from A to S₁ : $x = 2 \sin 32 = 1.060$ & $y = 2 \cos 32 = 1.696$ distances from A to S₂ : $x = 3 \sin 80 = 2.954$ & $y = 3 \cos 80 = 0.521$ so distances from S₁ to S₂ : $x = 1.894$ & $y = 1.175$ distance between S₁ to S₂ : $\sqrt{x^2 + y^2} = 2.229$ angle: 31.81° so 2.23 km at bearing 122°	any two correct distances from A	2.23 km at bearing 122°	
(e)	original surface area $A = 20^2 + 4 \times 15 \times 20 = 2000 \text{ cm}^2$ new block surface area, starting by finding x which is on a triangle with sides $20 - 12 = 8$ and $20 - 8 = 12$ so $x^2 = 12^2 + 8^2 = 14.42$ top and bottom area $20^2 - \frac{1}{2} \times 12 \times 8 = 352$ perimeter of top: $P = 20 + 20 + 12 + 8 + x = 74.42$ area of sides = $15P = 1116.33$ total surface area of prism $1116.33 + 2 \times 352 = 1820.33$ percentage of surface area $\frac{1820.33}{2000} \times 100\% = 91\%$	one of surface area of original block = 2000 OR $x = 14.42$	surface area of top $A = 352$	MERIT and percentage

(f)	<p>The shaded shape has angle $x = 90$ The area is</p> $2\pi = \frac{\pi \times 90 \times 5^2}{360} - \frac{\pi \times 90 \times R^2}{360}$ $2\pi = \frac{25\pi}{4} - \frac{R^2\pi}{4}$ $\frac{R^2\pi}{4} = \frac{25\pi}{4} - 2\pi$ $\frac{R^2}{4} = \frac{25}{4} - 2$ $R^2 = 25 - 8 = 17$ $R = \sqrt{17}$ $R = 4.123 \text{ m}$	<p>area of the whole sector $\frac{25}{4}\pi$</p> <p>to highlighted equation</p>		answer with algebraic working 4.123 m
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Each Question

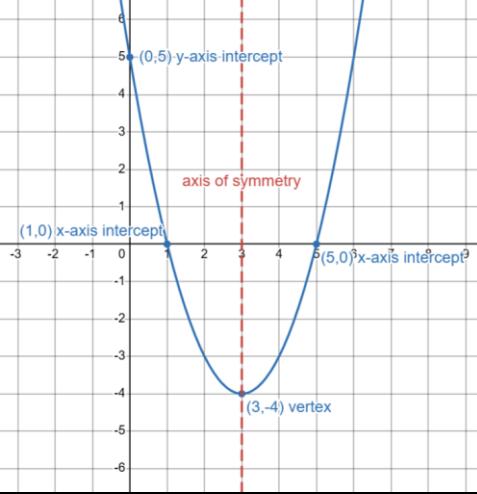
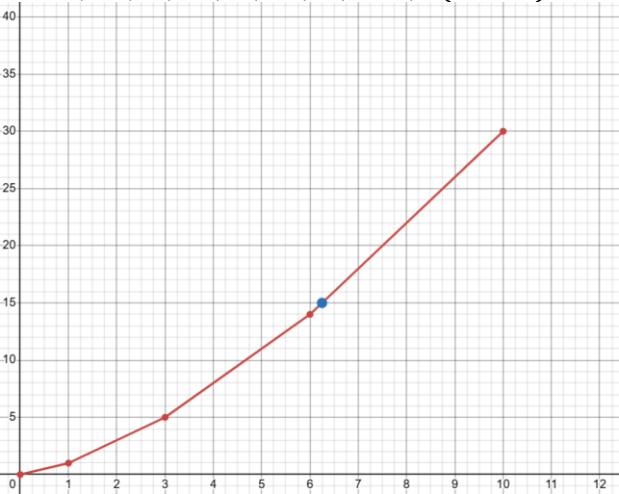
no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8

Total

0	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24
not achieved	nearly achieved	low achieved	high achieved	low merit	high merit	low excellence	high excellence	
NOT ACHIEVED			ACHIEVED			MERIT		EXCELLENCE
0-6			7-12			13-18		19-24

Evidence Statement

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$2x + 7 \leq 4x + 17$ $-2x \leq 10$ $x \geq -5$	some working required		
(b)	$m = \frac{\text{rise}}{\text{run}} = \frac{1-5}{4-1} = -\frac{4}{3}$ $y = -\frac{4}{3}x + c$ <p>substitute (1,5)</p> $5 = -\frac{4}{3} \times 1 + c \text{ so } c = \frac{19}{3}$ $y = -\frac{4}{3}x + \frac{19}{3}$ <p>or $y - 5 = -\frac{4}{3}(x - 1)$</p> <p>or $y - 1 = -\frac{4}{3}(x - 4)$ etc</p> $\frac{4}{3}x + y = \frac{19}{3}$ $4x + 3y = 19$	any form of equation	final answer (or a multiple with integers)	

(c)	$50 = 80 - 5t$ $-30 = -5t$ $t = \frac{-30}{-5} \text{ so } t = 6 \text{ minutes}$	algebra required in answer		
(d)		<p>correct shape, and two of the labelled features</p> <p>correct shape and ALL of the labelled features</p>		
(e)	$2x^2 + 6x + 5x + 15 + 2x = 0$ $2x^2 + 13x + 15 = 0$ $(2x + 3)(x + 5) = 0$ <p>either $x = -1.5$ or $x = -5$</p> <p>the vertex is half-way between these roots at $x = -3.25$</p> $y = (2 \times -3.25 + 3)(-3.25 + 5)$ $y = -6.125$	<p>expand and simplify</p> <p>factorise and roots</p>		<p>algebraic working and minimum</p>
(f)	<p>The insect has positions $(0,0), (1,1), (3,5), (6,14), (10,30)$</p>  <p>so the half-way distance $d = 15 \text{ cm}$ on the line between $(6,14)$ and $(10,30)$.</p> <p>We are given $m = 4$, $d = 4(t - 14)$ rearranges to $d = 4t - 10$</p> $15 = 4t - 10$ $4t = 25$ $t = 6.25$	<p>correct graph</p>	<p>equation found for fourth line $y = 4t - 10$ (any form)</p>	<p>algebraic working to get: $d = 15$ gives $t = 6.25$ seconds</p>

Evidence Statement

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	common ratio $r = 3$ $T_n = 2 \times 3^{n-1}$	equation for T_n		
(b)	$x^2 + (x + 7)^2 = (x + 8)^2$ $x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$ $x^2 - 2x - 15 = 0$ $(x - 5)(x + 3) = 0$ either $x = -3$ (reject as not a triangle) or $x = 5$ cm (answer)	quadratic formed and simplified	$x = -3$ rejected, $x = 5$ cm	
(c)	Let a be the age of Andrea and b be the age of Blake $b = a + 6$ $b = 1.15a$ then $1.15a = a + 6$ $0.15a = 6$ $a = 6 \div 0.15 = 40$ $n = a + 6 = 46$ Andrea is 40 and Blake is 46	equations formed	solution in context	
(d)	$b = 1.1h$ $A = \frac{2}{3}bh = \frac{2.2}{3}h^2 = 165$ $h^2 = 165 \times \frac{3}{2.2} = 225$ $h = 15$ cm $b = 16.5$ cm	equation formed $\frac{2.2}{3}h^2 = 165$	uses algebra: both dimensions $h = 15$ $b = 16.5$	
(e)	$A = 2$ and $B = 2$ $4^x - 2 = 2$ gives $4^x = 4$ so $x = 1$, the point is $(1,2)$ $4^{x-2} = 2$ gives $(2^2)^{x-2} = 2^1$ $2^{2x-4} = 2^1$ $2x - 4 = 1$ $x = 2.5$ the point is $(2.5,2)$	$A = 2$ AND $B = 2$	AND point $(1,2)$	AND algebraic working for point $(2.5,2)$
(f)	applying similar triangles to the cones: cone 1 with height H has radius x has $V_1 = \frac{1}{3}\pi x^2 H$ cone 2 with height $2H$ and radius $2x$ has $V_2 = \frac{1}{3}\pi(2x)^2 \times 2H = \frac{8}{3}\pi x^2 H$ whole cone with height $3H$ and radius $3x$ has $V_3 = \frac{1}{3}\pi(3x)^3 \times 3H = \frac{27}{3}\pi x^2 H$ Bottom piece has volume $P_3 = \left(\frac{27}{3} - \frac{8}{3}\right)\pi x^2 H$ $P_3 = 19 \times \frac{1}{3}\pi x^2 H = 19V_1$ $P_1:P_3 = 1:19$	V_2 or V_3	volume of P_3 $\frac{19}{3}\pi x^2 H$	correct ratio fully simplified