

Mathematical Reasoning: Practice Exam

<https://sites.google.com/view/snedomaths/>

AS 91947

1

1.4 Demonstrate Mathematical Reasoning

Total

0	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24
not achieved	nearly achieved	low achieved	high achieved	low merit	high merit	low excellence	high excellence	
NOT ACHIEVED			ACHIEVED			MERIT		EXCELLENCE
0-6			7-12			13-18		19-24

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)												
(a)	$\frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{1}{3}r$ $c = \frac{1}{3}$	working shown clear value of $c = \frac{1}{3}$														
(b)	<p>Call the numbers F and S</p> $F + 2S = 77$ $2F + S = 100$ <p>double first equation</p> $2F + 4S = 154$ <p>subtract second equation</p> $3S = 54 \text{ so } S = 18$ <p>substituting back, $F + 2 \times 18 = 77$</p> $F = 77 - 36 \text{ so } F = 41$	correct equations formed	algebraic working both numbers found													
(c)	<table border="1"> <tr> <td>65</td><td>1.25</td><td>52</td></tr> <tr> <td>240</td><td>6</td><td>40</td></tr> <tr> <td>190</td><td>2.75</td><td>69.09</td></tr> <tr> <td>495</td><td>10</td><td>49.5</td></tr> </table>	65	1.25	52	240	6	40	190	2.75	69.09	495	10	49.5	<p>all yellow correct</p> <p>all yellow and green correct</p> <p>all numbers correct, and correct graph (points not required)</p>		
65	1.25	52														
240	6	40														
190	2.75	69.09														
495	10	49.5														

	Expect Coverage	Achievement (u)	Merit (r)	Excellence (t)
(d) (i)	<p>expanding:</p> $A = 90 + 30x + 72x + 24x^2 - 90$ $A = 24x^2 + 102x$ <p>expanding:</p> $B = 24x^2 + 102x$	expand either expression correctly	expand both expressions correctly	
(d) (ii)	<p>expanding $(3x + 15)(8x - 6)$</p> $= 24x^2 - 18x + 120x - 90$ $= 24x^2 + 102x - 90$ <p>There are two roots, either</p> $3x + 15 = 0$ $3x = -15$ $x = -5$ <p>or</p> $8x - 6 = 0$ $8x = 6$ $x = 0.75$ <p>If the distance x is 0.75m then the total area of the surrounding rectangle is</p> $(5 + 4 \times 0.75) \times (18 + 6 \times 0.75)$ $= 8 \times 22.5 = 180$	<p>correctly expands or factorises to show last line is correct</p> <p>OR</p> <p>finds one or both values of x</p>	<p>correctly expands or factorises to show last line is correct</p> <p>AND</p> <p>finds BOTH values of x</p>	<p>eliminates $x = -5$ and gives $x = 0.75$m as final answer</p>

Each Question

no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8

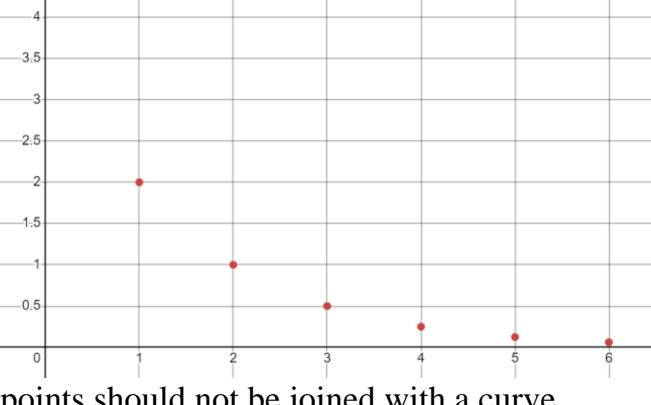
Evidence Statement

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$Y = abd \left(\frac{A}{a} + \frac{B}{b} + \frac{C}{c} \right)$ $= \frac{Aabd}{a} + \frac{Babd}{b} + \frac{Dabd}{d}$ $= Abd + aBd + abD$	correct answer note: answer given		
(b)	<p>Since $28^2 + 96^2 = 10\ 000 = 100^2$, the triangle satisfies the Pythagoras' Theorem.</p> <p>Using trigonometry, for example:</p> <p>The smaller angle is $\sin^{-1} \left(\frac{28}{100} \right) = 16.26^\circ$</p> <p>The larger angle is $\sin^{-1} \left(\frac{96}{100} \right) = 73.64^\circ$</p> <p>(or by $16.26 + 73.64 + 90 = 180$)</p>	Pythagoras used correctly	using trig, both angles found (90° not needed)	
(c)	<p>area of circles = $3 \times \pi \times 1^2 = 3\pi = 9.425$</p> <p>base of triangle $b = 4.732 - 0.732 = 5.464$</p> <p>height of triangle $h = 4.732$</p> $A = \frac{1}{2}bh = 12.928$ <p>Area not covered = $12.928 - 9.425 = 3.503 \text{ cm}^2$</p>	area of circles or area of equilateral triangle found	working shown, both areas found, and final answer	
(d)	<p>One end circle area: $A = \pi \times 8^2 = 64\pi$</p> <p>This is 5% of total surface area, so:</p> <p>Total surface area is $64\pi \div 0.05 = 1280\pi$</p> <p>Total surface area calculation from formula:</p> $2\pi r^2 + 2\pi rh =$ $2\pi \times 8^2 + 2\pi \times 8 \times h = 1280\pi$ $128\pi + 16\pi h = 1280\pi$ $16\pi h = 1152\pi$ $h = 72 \text{ cm}$ <p>Volume</p> $V = \pi r^2 h = \pi \times 8^2 \times 72 = 14\ 476 \text{ cm}^3$ <p>The container does hold (well over) 4 litres.</p>	total surface area calculated $1280\pi = 4021.2$	correct value of $h = 18$	volume calculated and checked, much more than 4 litres

(e)	<p>Weight of bread (two slices) $55\% \times 2 \times 38 = 41.8$ g</p> <p>Proportion of sandwich is bread by weight $\frac{41.8}{41.8 + 24} = 0.635$</p> <p>Volume of bread (two slices) $70\% \times 2 \times 135 \times 0.9 = 170.1$ cm²</p> <p>Volume of cheese $9^2 \times 0.3 = 24.3$ cm²</p> <p>Proportion of sandwich is bread by volume $\frac{170.1}{170.1 + 24.3} = 0.875$</p> <p>The sandwich is 63.5% bread by weight, and is 86.5% bread by volume.</p>	<p>at least one correct calculation of bread weight bread volume cheese volume</p>	<p>proportion of sandwich is bread by weight or volume</p>	<p>both proportions, compared</p>
-----	--	--	--	-----------------------------------

no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8

Evidence Statement

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a) (i)	Right angle triangle has sides 8mm and 17mm $\tan x = \frac{17}{8}$ so $x = 64.8^\circ$. The obtuse angle is $2x = 129.6^\circ$	129.6°		
(a) (ii)	There is a scale factor k , for which $k^2 = 0.2$ So $k = \sqrt{0.2} = 0.447$ The height of the inside triangle is $0.447 \times 34 = 15.2$ mm The width of the inside triangle is $0.447 \times 8 = 3.6$ mm	correct scale factor; $k = \sqrt{0.2}$ sufficient	both dimensions	
(b) (i)	The base is $2^2 \times 1 = 4$ cm So the angles are $\tan^{-1} 4 = 76^\circ$ and $\tan^{-1} 0.25 = 14^\circ$	base of first triangle	correct angles	
(b) (ii)	$A = 4 \times 0.5^n = 0.5^{n-2} = 2^{2-n}$  <p>points should not be joined with a curve</p>	table of values OR plot points (or curve) OR correct equation correct equation	plot points (or curve) AND correct equation AND correct equation	plot points NOT curve AND correct equation
(c)	expanding left side $x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 3x^2 + 6x + 5$ expanding right side $x^2 + 6x + 9 + x^2 + 8x + 16 = 2x^2 + 14x + 25$ equating $3x^2 + 6x + 5 = 2x^2 + 14x + 25$ collect terms $x^2 - 8x - 20 = 0$ factorising $(x + 2)(x - 10) = 0$ so $x = -2$ or $x = 10$ [check $10^2 + 11^2 + 12^2 = 365 = 13^2 + 14^2$]	expand two brackets correctly collect terms to single quadratic		solve and keep positive integer solution

no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8