

AS 91947

1.4 Demonstrate mathematical reasoning (5 credits)

You should attempt ALL the questions in this booklet.
The resource booklet 91947R should be with this booklet.
Show ALL working.
An approved calculator is allowed for this assessment.

Achievement	Achievement with Merit	Achievement with Excellence	Score	Grade
Demonstrate mathematical reasoning.	Demonstrate mathematical reasoning with relational thinking.	Demonstrate mathematical reasoning with extended abstract thinking.		

Grading information

Each Question

no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8

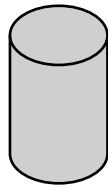
Total

0	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24
not achieved	nearly achieved	low achieved	high achieved	low merit	high merit	low excellence	high excellence	
NOT ACHIEVED			ACHIEVED			MERIT		EXCELLENCE
0-6			7-12			13-18		19-24

QUESTION ONE

(a) A cylinder is twice as tall as it is wide.

Write an expression for the volume in terms of the radius r , and simplify it where possible.



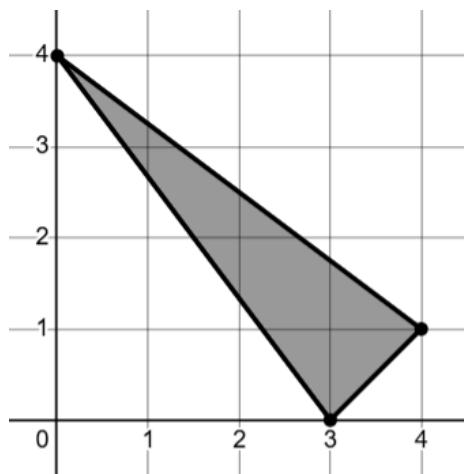
(b) A rectangular poster has dimensions 40 by 60 cm. A uniform border of width x is left blank around the image on the poster.



Write an expression for the area of the image in terms of x .

(c) Find the perimeter of the triangle shown in the diagram.

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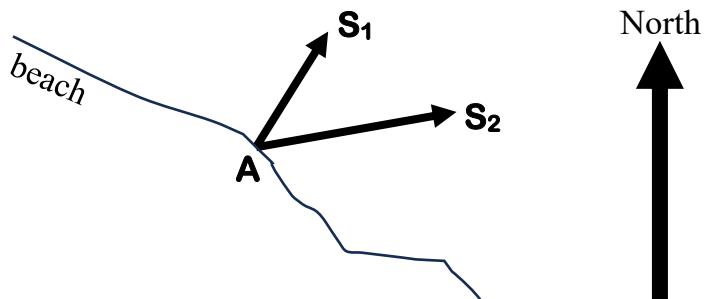


(d) Astoria stands on a beach at point **A** and can see two ships.

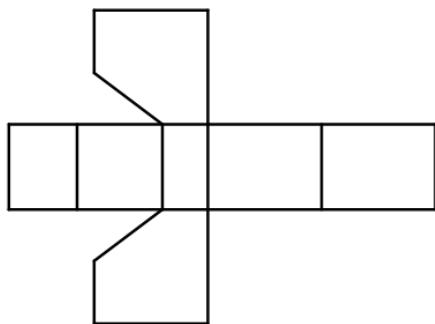
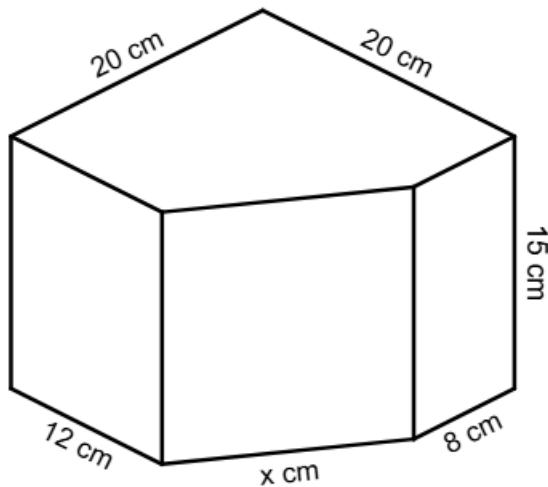
The first ship (**S₁**) is 2km away at a bearing of 032°.

The second ship (**S₂**) is 3km away at a bearing of 080°.

Find the distance between the two ships, and the bearing from \mathbf{S}_1 to \mathbf{S}_2 .



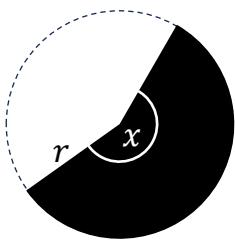
(e) A rectangular block starts 20 by 20 by 15 centimetres, and has a side cut off to make the prism below. Its net is shown to the right.



Find the percentage that the surface area of the block has been reduced.

(f) The area of a sector of a circle is calculated with the formula

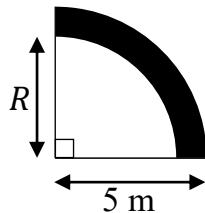
$$A = \frac{\pi x r^2}{360}$$



where r is the radius, and x is the angle (in degrees) at the centre of the circle.

The area of the shaded shape shown below is 2π m².

Find the inner radius R .



QUESTION TWO

(a) Solve the inequality $2x + 7 \leq 4x + 17$.

(b) A straight line passes through (1,5) and (4,1).

Use algebraic methods to write an equation for the line through these points.

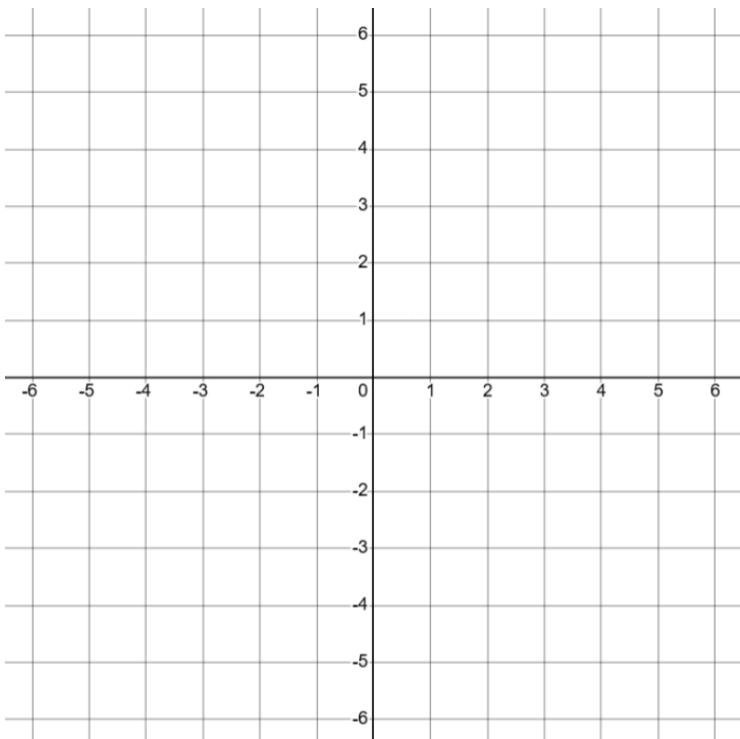
Write your answer in the form $ax + by = c$ where a , b and c are integers.

(c) A simple model for the temperature T °C of a liquid is $T = 80 - 5t$, where t is the time in minutes after it is removed from a heating element.

Use algebraic reasoning to find when the temperature reaches 50 °C.

(d) Sketch the parabola $y = x^2 - 6x + 5$, labelling

- the vertex
- the axis of symmetry
- the x -axis intercepts
- the y -axis intercept



(e) Solve $(2x + 5)(x + 3) + 2x = 0$.

Use these x -values to find the **smallest** value of $y = (2x + 5)(x + 3) + 2x$.

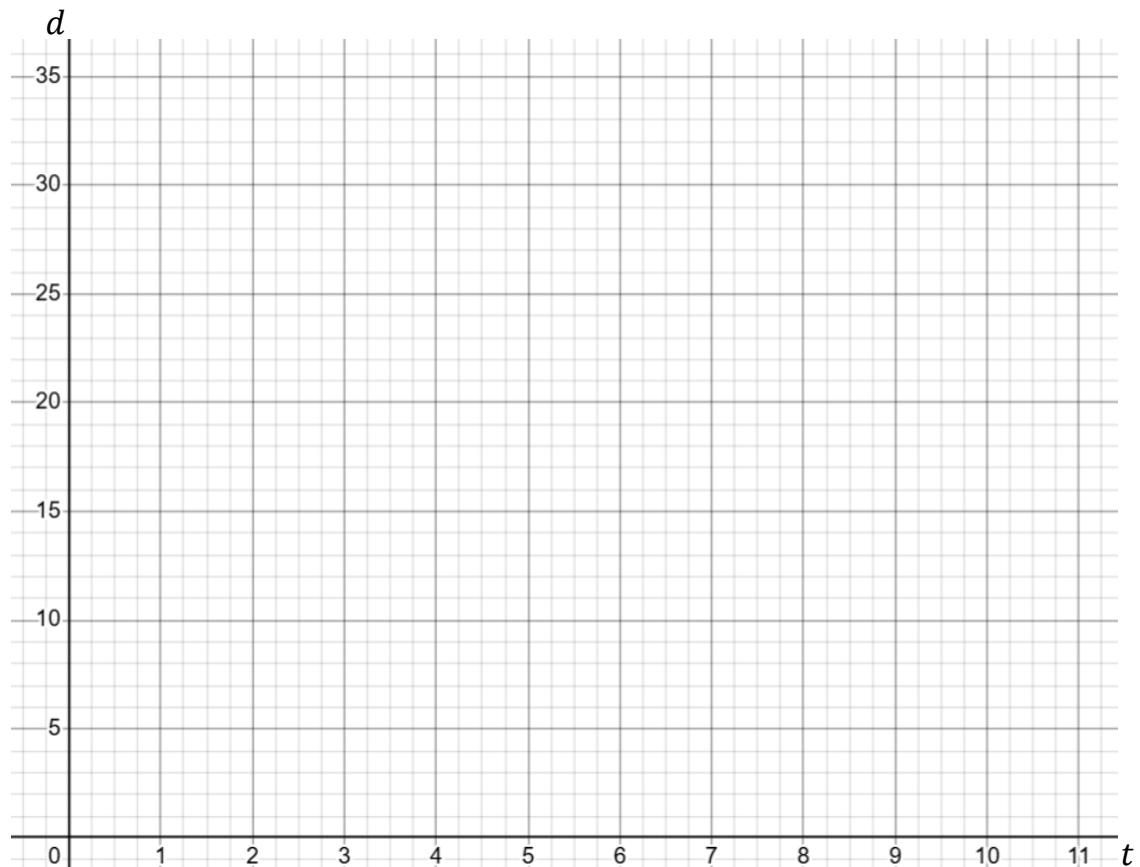
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(f) A small insect travels north for 1 second at 1 cm per second,
then for 2 seconds at 2 cm per second,
then for 3 seconds at 3 cm per second,
and finally for 4 seconds at 4 cm per second.

At time $t = 0$, the insect's position is $d = 0$.

- Sketch the position of the insect, d , at time t on the axes below.
- Find the time when the insect has travelled half the total distance.

Show algebraic working in your answer.



QUESTION THREE

(a) The first four terms in a sequence are 2,6,18,54, ...

Find an equation for the n th term in the sequence.

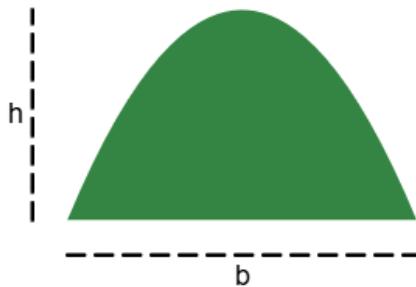
(b) The sides of a right-angled triangle are x , $x + 7$ and $x + 8$ cm.

Form a quadratic equation and solve to find the value of x .

(c) Andrea is 6 years older than her husband Blake. She also says she is 15% older than Blake.

Write two equations that represent this information and solve for their ages.

(d) The area of a parabolic segment can be calculated with the equation $A = \frac{2}{3}bh$ where b is the base width and h is the height.



A parabolic segment has area 165 cm^2 , and the base is 10% wider than the height.

Find the dimensions b and h .

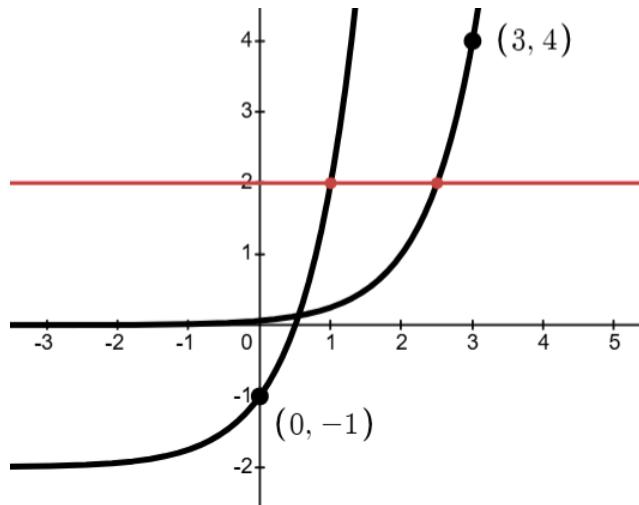
Show algebraic working.

e) The graph below shows the exponential curves

$$y = 4^x - A$$

$$y = 4^{x-B}$$

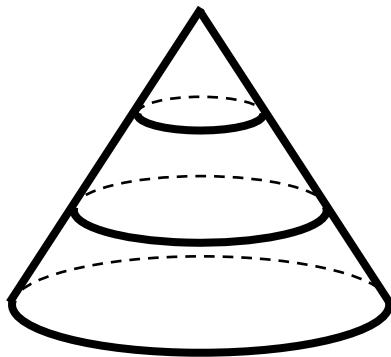
and the horizontal line $y = 2$.



Find the values of A and B , and the points where the horizontal line intersects with the two exponential curves.

(f) A cone of height $3H$ is cut horizontally to form three pieces as shown. Each piece has height H .

Use a radius of $3x$ for the base of the whole cone.



Find the ratio of the volume of the smallest piece to the volume of the largest, in the simplest form.