Mathematical Reasoning: Practice Exam

https://sites.google.com/view/snedmaths/new-level-1

1

AS 91947

1.4 Demonstrate Mathematical Reasoning (5 credits)

You should attempt ALL the questions in this booklet.
The resource booklet 91947R should be with this booklet.
Show ALL working.
An approved calculator is allowed for this assessment.

YOU MUST HAND THIS WORKBOOK TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement	Achievement with Merit	Achievement with Excellence	Score	Grade
Demonstrate mathematical reasoning.	Demonstrate mathematical reasoning with relational thinking.	Demonstrate mathematical reasoning with extended abstract thinking.		

Grading information

Each Question

no attempt	relevant attempt	1u	2u	3u	1r	2r	1t	2t
N0	N1	N2	A3	A4	M5	M6	E7	E8

Total

0	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24
not achieved		nearly achieved	low achieved	high achieved	low merit	high merit	low excellence	high excellence
NOT ACHIEVED			ACHIEVED		MERIT		EXCELLENCE	
0-6			7-12		13-18		19-24	

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Show that the ratio of the volume of a sphere of radius r to its surface area,

$$\frac{\text{volume}}{\text{surface area}}$$

can be simplified to $c \times r$, where c is a constant.

State clearly the value of c.

(b) Frankie and Sam each think of a number.

Frankie's number plus twice Sam's number is 77.

Sam's number plus twice Frankie's number is 100.

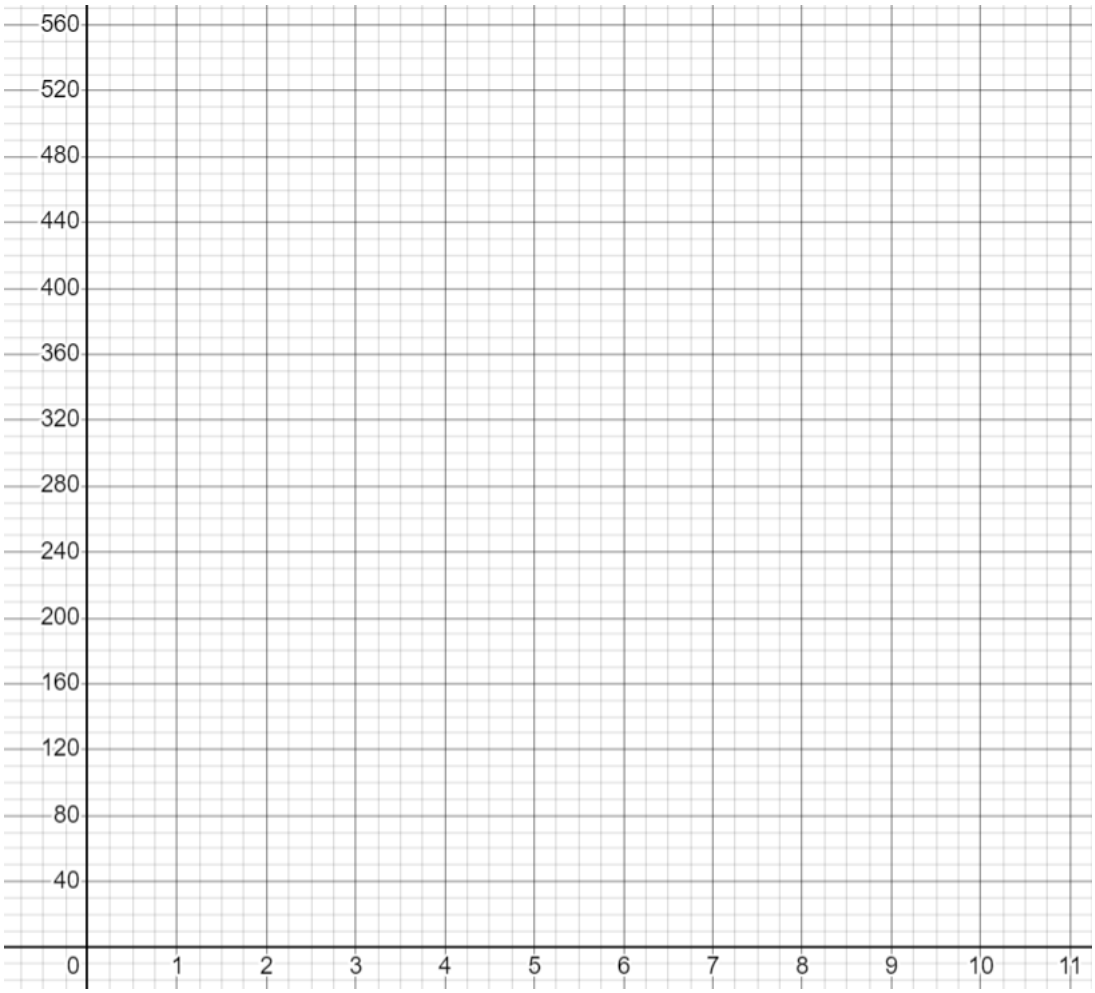
Use algebra to find their numbers.

ASSESSOR'S USE ONLY

(c) Victor keeps partial records of his distance travelled, time travelled, and average speed on three parts of a trip.

section of trip	distance (km)	time	average speed (km/h)
Tauranga to Rotorua		1 hour 15 minutes	52 km/h
Rotorua to Napier	240 km		40 km/h
Napier to Palmerston North	190km	2 hours 45 minutes	
whole trip:			

Complete the table above and distance-time graph below.

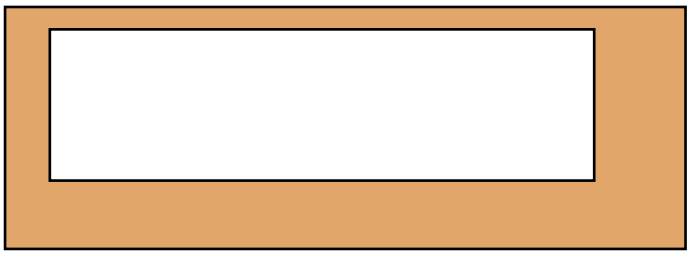


ASSESSOR'S USE ONLY

(d) The paving around a garden changes width on each side, given in the table below.

The central garden is 5 metres by 18 metres (total 90 square metres).

side:	top	left	bottom	right
width:	x	$2x$	$3x$	$4x$



(i) Ash writes the area of paving as $A = (5 + 4x)(18 + 6x) - 90$.

Benji write the area of paving as $B = 6x(4x + 17)$.

Show that these two representations are equal.

ASSESSOR'S USE ONLY

(ii) Carol wants to find width x which makes the shaded area of paving also have an area of 90 m^2.

She does the following algebraic working:

$$6x(4x + 17) = 90$$
$$24x^2 + 102x - 90 = 0$$
$$(3x + 15)(8x - 6) = 0$$

Show that her last line of working (the factorisation) is correct, and give the width x for her final answer.

ASSESSOR'S USE ONLY

QUESTION TWO

(a) A mathematical problem involves the following two equations.

$$y = abd$$

$$Y = y\left(\frac{A}{a} + \frac{B}{b} + \frac{D}{d}\right)$$

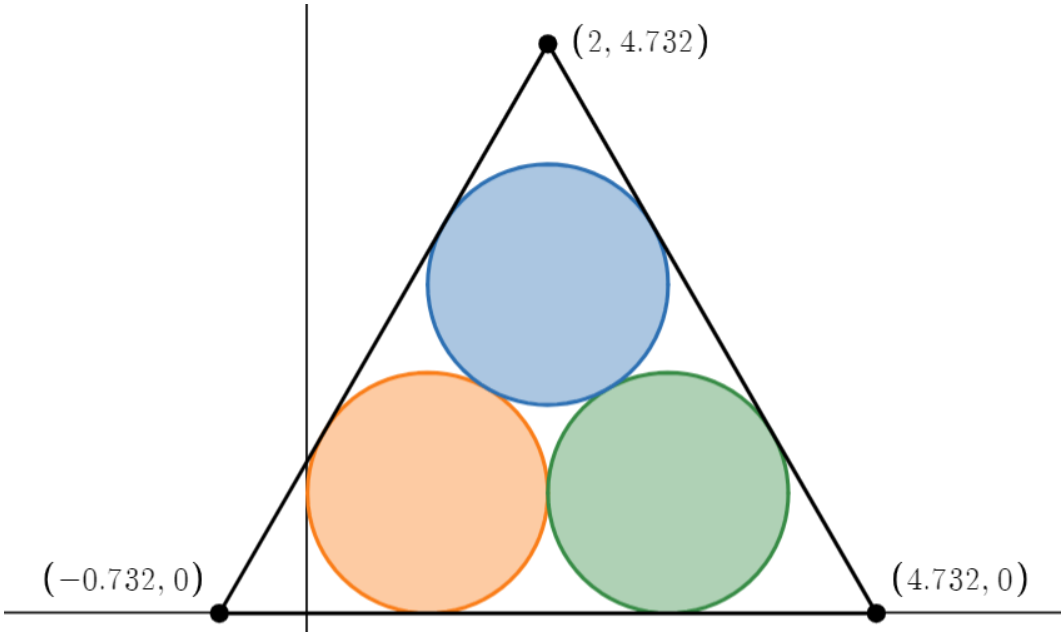
Show that these can be rearranged to $Y = Abd + aBd + abD$.

Be careful with uppercase and lowercase letters.

(b) Explain why a triangle with side lengths 28 mm, 96 mm and 100 mm is a right angled triangle, then calculate the angles at its corners.

ASSESSOR'S USE ONLY

(c) The diagram below shows an equilateral triangle, with three circles of radius 1cm drawn inside. The (x, y)-coordinates of the triangle corners are given.



Calculate the area of the triangle not covered by the shaded circles.

ASSESSOR'S USE ONLY

(d) Josh is designing a cylindrical container that is closed at both ends with radius 8 cm.

He needs the area of one of the end circles to be 5% of the total surface area of the container.

Check whether the container can hold 4 litres.

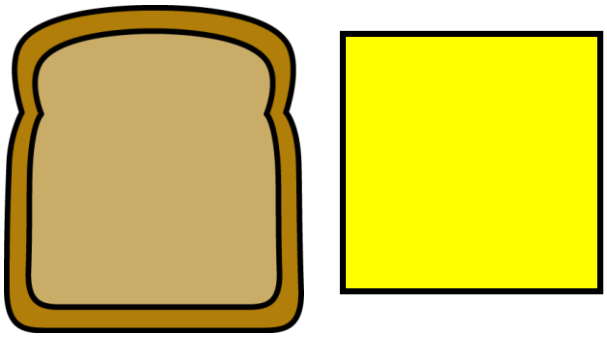
ASSESSOR'S USE ONLY

(e) The image below shows a slice of bread and a slice of cheese.

The total cross-sectional area of the bread is 135 cm^2, and it weighs 38 grams. It is 9 mm thick. The outer crust makes up 30% of the bread by volume, and 45% of the bread by weight.

Also shown is 24 gram slice of cheese. The cheese is a 9 cm square, and is 3 mm thick.

The image is only approximately to scale.



Ivan's mother makes him a sandwich with two slices of bread that have the crusts removed and one slice of cheese.

Show that calculating the proportion of the sandwich which is bread by **weight** and by **volume** give quite different answers.

ASSESSOR'S USE ONLY

QUESTION THREE

(a) The logo for the band **Def Leppard** is shown below.

The outside of triangular letter D is an isosceles triangle 8 mm wide and 34 mm tall.

(i) Find the obtuse angle in the letter D.

(ii) The triangle inside the letter D has the same internal angles. It takes up 20% of the area of the triangle. Find the dimensions of the smaller triangle.

ASSESSOR'S USE ONLY

(b) The diagram below shows a sequence of triangles. All have the same height, while the base is half the previous length at each step.

The third triangle has base and height both 1cm.

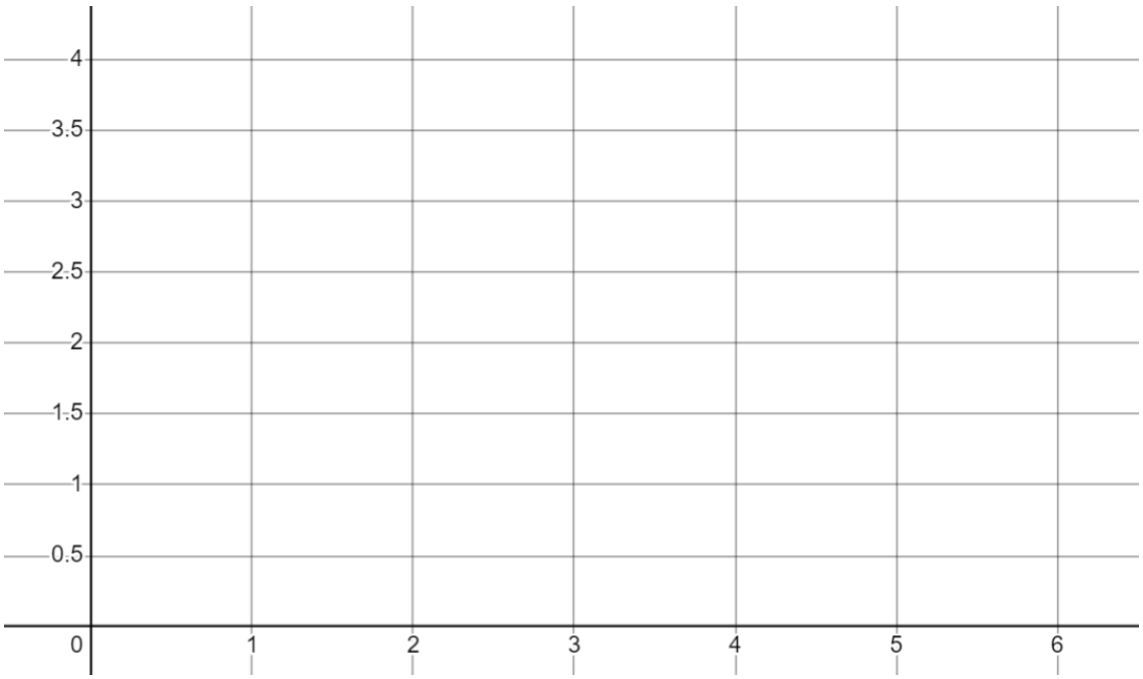
(i) Find the angles in the first triangle.

ASSESSOR'S USE ONLY

ii) Sketch a graph of the areas of the triangles, with n (the number of the triangle in the sequence) on the horizontal axis.

Give an equation for the area A in terms of n.

triangle number n	area A



ASSESSOR'S USE ONLY

(c) There are five consecutive positive integers that when squared and added satisfy the following equation. Here x represents the smallest integer.

$$x^2 + (x+1)^2 + (x+2)^2 = (x+3)^2 + (x+4)^2$$

Use algebra to find the value of x.